Effect of Heat Source/Sink and Mixed Convection Flow of Maxwell Fluid on Heat Transfer near A Stagnation Point with Convective Condition

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Abstract

The effects of thermal radiation and heat transfer of a Maxwell fluid near a mixed convection stagnation point flow over a moving surface in the presence of heat source/sink has been studied. The governing differential equations are transformed into a set of coupled non-linear ordinary differential equations and then solved with a numerical technique using appropriate boundary conditions for various physical parameters. The numerical solution for the governing non-linear boundary value problem is based on applying the fourth-order Runge-Kutta method coupled with the shooting technique using appropriate boundary conditions for various physical parameters. The effects of various parameters like the viscosity parameter, radiation parameter, mixed convection parameter, Deborah number, heat source/sink parameter and Prandtl number on the velocity and temperature profiles as well as on the local skin-friction coefficient and the local Nusselt number are presented and discussed.

Keywords: Mixed convection flow, Thermal radiation, Maxwell fluid, heat transfer, heat source/sink, Convective condition.

Introduction

There is no single constitutive relationship between stress and rate of strain by which all the non-Newtonian fluids can be examined. The main difficulty in researching a general boundary-layer theory for non-Newtonian fluids lies, obviously, in the diversity of these fluids, in their constitutive behaviour, simultaneous viscous and elastic properties such that differentiating between those effects which arise as a result of a fluid’s shear-dependent viscosity from those which are attributable to the fluid’s elasticity becomes virtually impossible. But some mathematical models have been proposed to fit well with the experimental observations [1]. The simplest model for the rheological effects of viscoelastic fluids is the Maxwell model where the dimensionless relaxation time is small. However, in some more concentrated polymeric fluids, the Maxwell model is also used for large dimensionless relaxation time. Vieru et al. [2] applied Fourier and Laplace transforms to find exact solutions of a fractional Maxwell model for flow between two-sided wall perpendiculars to a plate. Hayat et al. [3] analyzed the MHD flow and mass transfer of a UCM fluid past a porous shrinking sheet on the presence of chemical reaction species. Hayat et al. [4] studied the MHD unsteady flow of a Maxwell fluid in a rotating frame of reference and porous medium. It must be noted that the Maxwell fluid model allows for the relaxation effects which cannot be predicted in other different types of non-Newtonian fluids such as second, third and fourth grades. Stanford Shateyi [5] considered the steady MHD flow of a Maxwell fluid past a vertical stretching sheet in a Darcian porous medium.

Recently, the subject of hydromagnetics has attracted the attention of many authors due to its applications to problems which have geophysical and astrophysical significance. In modern metallurgical and metal-working processes, the study of MHD flow of an electrically conducting fluid is of considerable interest (Mukhopadhay [6]). To purify the molten metals from non-metallic inclusions, hydromagnetic techniques are used. This type of problem is very much useful to polymer technology and metallurgy. Hayat et al. [7] reported a series solution for MHD boundary layer flow of an upper convected Maxwell fluid over a porous stretching sheet. Mukhopadhay [8] reported the magnetic effects on unsteady non-isothermal stretching sheet assuming the temperature variation
with space and also with time. Bhattacharyya et al. [9] discussed the MHD boundary layer flow for diffusion of a chemically reactive species. In this paper, they also analyzed the combined effects of unsteadiness and suction/blowing in presence of first order constructive/destructive chemical reaction. Sharidan et al. [10] analyzed the unsteady flow and heat transfer over a stretching sheet in a viscous and incompressible fluid by considering both the variable wall temperature (VWT) and variable heat flux (VHF) conditions. Tsai et al. [11] obtained solutions for unsteady flow and heat transfer over a stretching sheet and reported the effects of variable heat source/sink on heat transfer characteristics. Mukhopadhyay [12] obtained similarity solutions for unsteady mixed convection flow past a stretching sheet considering the combined effects of porous medium and thermal radiation. In that paper the variable permeability was assumed. Mukhopadhyay and Gorla [13] conclude that the effect of increasing values of the Maxwell parameter is to suppress the velocity field and the concentration is enhanced with increasing Maxwell parameter. Shateyi et al. [14] investigated the influence of a magnetic field on heat and mass transfer by mixed convection in the presence of Hall, radiation Soret and Dufour effects. Shateyi and Motsa [15] studied the effects of thermal radiation on heat and mass transfer over an unsteady stretching surface. Abel et al. [16] performed the analysis of the effect of MHD and thermal radiation on two-dimensional steady flow of an incompressible, upper converted Maxwell fluid.

The heat source/sink effects in thermal convection are significant where there may exist high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reaction. Tania et al [17] has investigated the Effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Vajravelu and Nayfeh [18] reported on the hydro magnetic convection at a cone and a wedge in the presence of temperature dependent heat generation or absorption effects. Swati Mukhopadhyay [19] analyzes the heat transfer analysis of the unsteady flow of a Maxwell fluid over a stretching surface in the presence of a heat source/sink.

The present study contains an analysis of the effects of mixed convective flow of a Maxwell fluid over a stretching sheet by taking heat source/sink into account. Using the similarity transformations, the governing equations have been transformed into a set of ordinary differential equations, which are nonlinear and cannot be solved analytically, therefore, fourth order Runge-Kutta method along with shooting technique has been used for solving it. The results for velocity and temperature functions are carried out for the wide range of important parameters namely, magnetic parameter, viscosity parameter, thermal conductivity parameter and radiation parameter. The skin friction and rate of heat transfer have also been computed.

2. Mathematical Formulation

Let us consider the two-dimensional mixed convection stagnation point flow of an incompressible and radiative Maxwell fluid near a stretched surface. The x-axis is taken along the stretching surface and y-axis perpendicular to the x-axis. The flow of an incompressible fluid is confined to \( y > 0 \). The thermo-physical properties of fluid and surface are taken constant. Under these assumptions along with the Boussinesq and boundary layer approximations, the system of equations, which models the flow is given by

Continuity equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}
\]

Linear momentum equation

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} + \lambda_1 \left[ u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right]
= u_e \frac{du}{dx} + \nu \frac{\partial^2 u}{\partial y^2} + g \beta_\gamma (T - T_\infty) \tag{2.2}
\]

Energy equation

\[
\frac{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}}{\rho c_p} = \frac{1}{\rho c_p} \left( k \frac{\partial T}{\partial y} \right) \frac{\partial q_r}{\partial y} + \frac{Q_r}{\rho c_p} (T - T_\infty) \tag{2.3}
\]

The boundary conditions for the velocity, temperature and concentration fields are

\[
u \frac{\partial u}{\partial y} = 0, \quad -k \frac{\partial T}{\partial y} = h(T_s - T) \quad \text{at} \quad y = 0
\]

\[
u \frac{\partial u}{\partial y} = \alpha \nu, \quad T \rightarrow T_s \quad \text{as} \quad y \rightarrow \infty \tag{2.4}
\]

where \( u \) and \( v \) are the velocity components along \( x \) and \( y \) directions, respectively, \( \rho \) is the fluid density, \( T \) is the temperature of the fluid, \( \lambda_1 \) is the relaxation time, \( c_p \) is the specific heat at constant pressure, \( \mu \) is the fluid viscosity, \( k \) is the fluid thermal conductivity, \( q_r \) is the radiative heat flux, \( Q_r \) is the heat source/sink.
constant, $T_\infty$ is the free stream temperature and $T_f$ is the convective fluid temperature.

By using the Rosseland approximation the radiative heat flux $q_r$ is given by

$$q_r = -\frac{4\sigma_s}{3k^*} \frac{\partial T^4}{\partial y}$$  \hspace{1cm} (2.5)

Where $\sigma_s$ is the Stefan–Boltzmann constant and $k^*$ is the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are significantly small, then equation [2.5] can be linearised by expanding $T^4$ into the Taylor series about $T_\infty$, which after neglect higher order terms takes the form:

$$T^4 \approx 4T_\infty^3 - 3T_\infty^2$$  \hspace{1cm} (2.6)

In view of equations (2.5) and (2.6), eqn. (2.3) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma_s T^3_c}{3k^* p c_p} \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (2.7)

$$+ \frac{Q}{p c_p} (T - T_e)$$

The continuity equation (2.1) is satisfied by the Cauchy Riemann equations

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$  \hspace{1cm} (2.8)

where $\psi(x, y)$ is the stream function.

In order to transform equations (2.2) and (2.7) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.

$$\psi = \sqrt{\nu x f(\eta)}, \eta = y \sqrt{\frac{1}{\nu}}, \theta = \frac{\theta}{\theta_0}, x = \frac{x}{\sqrt{\alpha}}$$

$$\beta(\eta) = \frac{T - T_e}{T_\infty - T_e}, \alpha = \frac{a}{c}, \beta = \frac{\lambda c}{\beta R}, R = \frac{4\sigma_s T^3_c}{k^* p c_p}, \lambda = \frac{Gr_f}{Re_f}$$

$$Q = \frac{\rho c_p}{\rho c_p} \frac{\beta R}{\nu^2} \frac{g}{\nu}, Pr = \frac{\nu}{\alpha}$$

where $f(\eta)$ is the dimensionless stream function, $\theta$ - dimensionless temperature, $\eta$ - similarity variable, $a$ and $c$ are constants, $\beta$ - Deborah number, $\lambda$ - mixed convection parameter, $\alpha$ - ratio of rate constant, $Gr_f$ - Grashof number, $Re_f$ - Reynolds number, $R$ - radiation parameter, $Pr$ - Prandtl number.

In view of Equations (2.8) - (2.9), the Equations (2.2) and (2.7) transform into

$$f'''' + \frac{ff''}{f} - f'' + \alpha^2 - \beta \left( f^2 f'' - 2ff' f' \right) + \lambda \theta = 0$$

The corresponding boundary conditions are:

$$f(0) = 0, f'(0) = 1, \theta'(0) = -\gamma(1 - \theta(0)) \hspace{1cm} (2.12)$$

$$f''(\infty) = \alpha, \theta'(\infty) = 0$$

where the primes denote differentiation with respect to $\eta$

The physical quantities of interest are the skin friction coefficient $C_f$, the local Nusselt number $Nu$, which are defined as

$$C_f = -2Re^{-1/2} f''(0), Nu = -Re^{1/2} \theta'(0)$$

## 3 Solution of the problem

The set of coupled non-linear governing boundary layer equations (2.10) and (2.11) together with the boundary conditions (2.12) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (2.10) and (2.11) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et al.[20]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta \eta = 0.05$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient and Nusselt number, which are respectively proportional to $f''(0)$ and $-\theta'(0)$, are also sorted out and their numerical values are presented in a tabular form.

## 4 Results and Discussion

The governing equations (2.10) - (2.11) subject to the boundary conditions (2.12) are integrated as described in section 3. In order to get a clear insight of the physical problem, the velocity and temperature have been discussed by assigning numerical values to the parameters encountered in the problem. The effects of various parameters on velocity profiles in the boundary layer are depicted in Figs. 1-7. The effects of various parameters on temperature profiles in the boundary layer are depicted in Figs. 8-13.
It is observed that the skin friction decreases as the viscosity parameter increases. Fig. 2 shows the variation of the velocity with the Deborah number (β). It is noticed that the velocity thickness increases with an increase in the Deborah number. Fig. 3 illustrates the effect of the convective parameter (γ) on the velocity field. It is seen that as the convective parameter increases, the velocity field increases. Fig. 4 illustrates the effect of the mixed convection parameter (λ) on the velocity field. It is seen that as the mixed convection parameter increases, the velocity field increases. Fig. 5 shows the variation of the thermal boundary-layer with the Prandtl number (Pr). It is noticed that the thermal boundary layer thickness decreases with an increase in the Prandtl number. Fig. 6 shows the variation of the velocity with the radiation parameter (R). It is noticed that the velocity thickness increases with an increase in the radiation parameter. Fig. 7 illustrates the effect of heat source/sink parameter (Q) on the velocity. It is noticed that as the heat source/sink parameter increases, the velocity increases.

Fig. 8 depicts the thermal boundary-layer with the viscosity parameter. It is noticed that the thermal boundary layer thickness decreases with an increase in the viscosity parameter. Fig. 9 depicts the thermal boundary-layer with the convective parameter. It is noticed that the thermal boundary layer thickness increases with an increase in the convective parameter. Fig. 10 illustrates the effect of the mixed convection parameter on the temperature. It is noticed that as the mixed convection parameter increases, the temperature decreases. Fig. 11 shows the variation of the thermal boundary-layer with the Prandtl number. It is noticed that the thermal boundary layer thickness decreases with an increase in the Prandtl number. Fig. 12 shows the variation of the thermal boundary-layer with the radiation parameter. It is observed that the thermal boundary layer thickness increases with an increase in the radiation parameter. Fig. 13 shows the variation of the thermal boundary-layer with the heat source/sink parameter. It is observed that the thermal boundary layer thickness increases with an increase in the heat source/sink parameter.

Table 1 show the variation of the skin friction and Nusselt number with for different values of α, β, γ, λ, R, Q and Pr. It is noticed that the skin friction increases where as Nusselt number decrease with an increase in the Deborah number. It is found that the skin friction decreases where as Nusselt number increase with an increase in the viscosity parameter or convective parameter or mixed convective parameter. It is observed that both the skin friction and Nusselt number increases with an increase in the Prandtl number. It is observed that both the skin friction and Nusselt number decreases with an increase in the radiation parameter or heat source/sink parameter. The correctness of the present numerical method is checked with the results obtained by Pop et al. [20], Mahapatra and Gupta [21] and Hayat et al. [22] for the values of Skin friction coefficient in the limiting condition. Thus, it is seen from Table 2.

### 5 Conclusions

The effects of thermal radiation and heat transfer of a Maxwell fluid near a mixed convection stagnation point flow over a moving surface in the presence of heat source/sink has been studied. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that:

1. The velocity increases as well as temperature decreases with an increase in the viscosity parameter.
2. The velocity and temperature increases with an increase in the radiation parameter or heat source/sink parameter.
3. The skin friction reduces the viscosity parameter or radiation parameter or heat source/sink parameter and increases with the Deborah number.
4. The Nusselt number reduces the Deborah number or radiation parameter or heat source/sink parameter and increases with the viscosity parameter.

![Fig.1 Velocity for different values of α](http://www.ijesrt.com)
Fig. 2 Velocity for different values of $\beta$

Fig. 3 Velocity for different values of $\gamma$

Fig. 4 Velocity for different values of $\lambda$

Fig. 5 Velocity for different values of $Pr$

Fig. 6 Velocity for different values of $R$

Fig. 7 Velocity for different values of $Q$
Fig. 8 Temperature for different values of $\alpha$  

Fig. 9 Temperature for different values of $\gamma$  

Fig. 10 Temperature for different values of $\lambda$  

Fig. 11 Temperature for different values of $Pr$  

Fig. 12 Temperature for different values of $R$  

Fig. 13 Temperature for different values of $Pr$
Table 1 Numerical values of $-f''(0), -\theta'(0)$ at the sheet for different values of $\alpha, \beta, \gamma, \lambda, R, Pr, Q$ and $Pr$

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<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\lambda$</th>
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<th>$Pr$</th>
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Table 2 Numerical values of $f''(0)$ at the sheet for different values of $\lambda$, Comparison of the present results with that of Pop et al. [21], Mahapatra and Gupta [22] and Hayet al.[23]

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References