Supply Quota Allocation and its optimization in supply chain by Fuzzy programming Model

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Abstract
Appropriate vendor selection and their optimum quota allocation has been an area of high importance in the effective management of a supply chain. The optimization of vendor-base is needed to identify better performing vendors in a supply chain. Secondly, their quota allocations need to be optimized for the organization to remain competitive in the global scenario.

The Supplier Quota Allocation (SQA) concept ‘Multi-Objective Linear Programming Vendor Selection Problem the mathematical formulation of which incorporates three important goals- cost- minimization, rejection-minimization (or quality-maximization) and minimization of late-delivery- with practical constraints imposed on: meeting the manufacturing organization’s annual aggregate demand, vendors’ capacity, vendors’ quota flexibility, vendors’ rating etc. In FP formulation of a supply chain modeling, various input parameters are treated as vague with a linear membership function of fuzzy type.

Keywords: supply chain management, fuzzy programming, LINDO, MATLAB.

Nomenclature
AHP : Analytical Hierarchy Process
DSS : Decision Support System
JIT : Just In Time.
MCD : Multiple Criteria Decision
QC : Quality Control
SC : Supply Chain
SCM : Supply Chain Management
VSP : Vendor Selection Problem
VDP : Vendor Development Program

Introduction
The supply chain of a manufacturing organization consists of all the activities, which are required to deliver the products and services to the end customers. It includes receiving orders from customers through marketing division, procuring raw materials from vendors, manufacturing, and logistics in man-machine-material management, marketing, customer relations and so on. The effective integration of information and material flow within the demand and supply process is what supply chain management is all about. “The objective of managing the supply chain is to synchronize the requirements of the customers with the flow of materials from suppliers in order to strike a balance between what are often seen as conflicting goals of high customer service, low inventory, and low unit cost”. Each node in a supply chain is a strategic link. The strong links make strong supply chains while the weak links hurt every member of the chain.

Appropriate vendor selection and their optimum quota allocation has been an area of high importance in the effective management of a supply chain. This is due to
the compelling need to develop long-term strategic alliances with the vendors, because material and equipment supplied by the vendors play an important role in the management of a supply chain of any organization. Many other issues in the supply chain are directly or indirectly influenced by the proper selection of vendors. Having a policy of approved reliable vendors may lead to less number of vendors in a supply chain, whereas the selection of a large number of vendors may be done to minimize the risk associated with the purchase. Hence, the optimization of vendor-base is needed to identify better performing vendors in a supply chain. Secondly their quota allocations need to be optimized for the organization to remain competitive in the global scenario.

**Literature review**

Muralidharan, Anantharaman, & Deshmukh, (2002) - The operations of all the organizations within the supply chain depend on the selection of suppliers that align with the goals and mission of the buying organization.

Petersen, Frayer, & Scannell, (2000) - The ultimate purpose of supplier selection is to develop and maintain a competitive advantage in the marketplace. To successfully compete in the global marketplace of today, firms must meet or exceed the pace of rapidly changing technology while also lowering costs, increasing quality, and improving customer service at all stages of the value chain. For this purpose the technology involved in a firm can have a significant impact on competition.

Zadeh (1965) initiated the fuzzy set theory and suggested the concept of fuzzy sets as a possible way of improving the modeling of vague parameters. Bellman and Zadeh (1970) suggested fuzzy programming model for decisions in fuzzy environment. Zimmermann (1976, 1978) presented a fuzzy optimization technique to linear programming (LP) problem with single and multiple objectives. Later, a number of researchers developed and applied fuzzy theories in the various areas of engineering application, such as, artificial intelligence, robotics, image processing, pattern recognition etc.

Dickson's work - a benchmark in the supplier's selection

<table>
<thead>
<tr>
<th>1. Price/cost</th>
<th>13. Customer service</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Delivery</td>
<td>14. Repair service</td>
</tr>
<tr>
<td>3. Warranties and claims</td>
<td>15. Training aids</td>
</tr>
<tr>
<td>5. Operating controls</td>
<td>17. Performance history</td>
</tr>
<tr>
<td>6. Production facilities</td>
<td>18. Reputation and position</td>
</tr>
<tr>
<td>7. Technical capability</td>
<td>19. Amount of past business</td>
</tr>
<tr>
<td>8. Packaging ability</td>
<td>20. Labour relations record</td>
</tr>
<tr>
<td>10. Management and organization</td>
<td>22. Impression</td>
</tr>
<tr>
<td>11. Communication system</td>
<td>23. Reciprocal arrangements</td>
</tr>
<tr>
<td>12. Desire for business</td>
<td></td>
</tr>
</tbody>
</table>


**Objective**

A lot of research work has been done in the field of selection, monitoring and maintaining appropriate vendors. But only little work has been done in allocating optimum quota to selected optimum number of vendors and that too in an uncertain environment with vague information. Now any vendor selection and quota allocation problem can be considered as a multi-objective problem with continuous variable in a fuzzy environment followed by its comparison with deterministic situation.

**Working Steps of Calculations and Applications:**

Input information collection and primitive comparative analysis.

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FP (Fuzzy Programming)
In traditional multiple-objective optimization all the information known with certainty and preciseness. But in the real-life situation for a Supplier Selection Problem, many input information related to the various vendors are not known with certainty. Due to this, here a fuzzy model has been considered.

Modeling the VSP problem with fuzzy parameters
[LPP] [fuzzy-LPP]

Minimize \( Z = Cx \)
Subject to \( \sim Cx \prec \sim Z_0 \),
\( Ax = b, \sim Ax \prec \sim b, \)
\( x = 0. \)

The symbol ‘\( \prec \)’ in the constraint set denotes ‘essentially smaller than or equal to’ and allows one reach some aspiration level where, \( \sim C \) and \( \sim A \) represent fuzzy values.

Membership Function
In solving the [fuzzy-SQA] model, a linear membership function has been considered in this work for all fuzzy parameters. A linear membership function has a continuously increasing or decreasing value over the range of parameter. It is defined by the lower and upper values of the acceptability for that parameter.

A fuzzy objective \( \sim Z \in X \) is a fuzzy subset of \( X \) characterized by its membership function \( \mu_Z(x) \): \( x \in [0, 1] \): The linear membership function for the fuzzy objectives is given as

\[
\mu_{S(x)} = \mu_{Z(x)} \cap \mu_{C(x)} = \min \left[ \mu_{Z(x)}, \mu_{C(x)} \right]
\]

Fuzzy Solution
In fuzzy programming modeling, using Bellman and Zadeh (1970) approach, a fuzzy solution is given by the intersection of the all the fuzzy sets representing either fuzzy objectives or fuzzy constraints. The membership function of the fuzzy solution is given by:

\[
\mu_S(x) = \mu_Z(x) \cap \mu_C(x) = \min [\mu_Z(x); \mu_C(x)]
\]

\( \mu_C(x) \) and \( \mu_S(x) \) represent the membership function of objectives, constraints and solutions, respectively.

The fuzzy solution of the [f-VSP] model for the \( L \) fuzzy multiple objectives and \( K \) fuzzy constraints may be given as

\[
\mu_S(x) = \bigcap_{l=1}^{L} \mu_{Z(l)} \cap \bigcap_{k=1}^{K} \mu_{C(k)}
\]

An optimum solution of the [f-VSP] is one which has the highest degree of the membership value:
Crisp formulation

\[
\begin{align*}
\text{Maximize } & \gamma \\
\text{Subject to } & \left[ \gamma (Z_l \max - Z_l \min) \right] + [Z_l(x)] \leq Z_l \max \\
& [\gamma (dx_l) + gk(xi)] \leq [bk + dk] \\
& Axi \leq b \\
& xi \geq 0 \\
& 0 \leq \gamma \leq 1 \\
& \text{for all } l; \ l = 1, 2, \ldots, L; \\
& \text{for all } k; \ k = 1, 2, \ldots, K; \\
& \text{All deterministic constraints.}
\end{align*}
\]

The lower bound of the optimal values (Zlmin) is obtained by solving the VSP as a linear programming problem using each time only one of the objectives (ignoring all others):

Minimize Zl (xi) for all l; l = 1, 2, \ldots, L; \\
Subject to gk(xi) = bk + dk for all k; k = 1, 2, \ldots, K; \\
Axi = b for all deterministic constraints; xi = 0

The upper bound of the optimal values (Zlmax) is obtained by solving a similar VSP as a linear programming problem:

Maximize Zl (xi) for all l; l = 1, 2, \ldots, L; \\
Subject to gk(xi) = bk + dk for all k; k = 1, 2, \ldots, K \\
Axi = b for all deterministic constraints; xi = 0

Application of [fuzzy -LPP] model to the SQA Model

The decision-maker’s ambiguity about the fuzzy information related to the net cost, rejections, late deliveries and vendors’ capacities is captured by applying the [fuzzy -LPP] model to the SQA model and fuzzified form [fuzzy -SQA] is expressed.

[Fuzzy -SQA] Model

\[
\begin{align*}
n & \Sigma p_i(x_i) \leq Z1, \\
i & = 1 \\
n & \Sigma q_i(x_i) \leq Z2, \\
i & = 1 \\
n & \Sigma l_i(x_i) \leq Z3, \\
i & = 1 \\
n & \Sigma x_i = D, \\
i & = 1 \\
x_i \leq U_i, \\
i & = 1 \\
n & \Sigma f_i(x_i) \geq P, \\
i & = 1 \\
n & \Sigma f_i(x_i) \leq F, \\
i & = 1 \\
p_i(x_i) \leq B_i, \\
x_i \geq 0
\end{align*}
\]

Application of Fuzzy Programming (FP) to case study SQA problem with fuzziness captured in capacities and allocated budgets of suppliers:

According to the theory, the linear membership function is used for fuzzifying the right-hand side of the constraints in the basic SQA model. The values of the level of uncertainties for all the fuzzy parameters are taken in incremental steps of 5% of the deterministic model. In order to find the values of cost, rejections, late deliveries at the lowest and highest level of membership function. The model is solved using LINDO – with first minimization and then maximization of only one objective at a time, ignoring all the other objectives. The data set for the values at the lowest and highest aspiration levels of the membership functions with 5% uncertainty solution is given by the intersection of the all the
Formulation of SQA Problem with 5% increase in vendor’s capacities and budget.

<table>
<thead>
<tr>
<th>MINIMIZE or MAXIMIZE</th>
<th>40000X1 + 33000X2 + 35000X3 + 32000X4</th>
</tr>
</thead>
<tbody>
<tr>
<td>MINIMIZE or MAXIMIZE</td>
<td>0.05X1 + 0.03X2 + 0.05X3 + 0.10X4</td>
</tr>
<tr>
<td>MINIMIZE or MAXIMIZE</td>
<td>0.05X1 + 0.03X2 + 0.08X3 + 0.04X4</td>
</tr>
<tr>
<td>SUBJECT TO:</td>
<td>X1 + X2 + X3 + X4 = 12000</td>
</tr>
<tr>
<td></td>
<td>X1 &lt;= 660.5</td>
</tr>
<tr>
<td></td>
<td>X2 &lt;= 2100</td>
</tr>
<tr>
<td></td>
<td>X3 &lt;= 1200</td>
</tr>
<tr>
<td></td>
<td>X4 &lt;= 2100</td>
</tr>
</tbody>
</table>

0.97X1+0.03X2=0.08X3+0.04X4 => 10920
0.04X1+0.03X2+0.08X3+0.04X4 <= 600
-4000X1 => -262000000
33000X2 => 105000000
33000X3 => 210000000
33000X4 => 630000000
X1, X2, X3, X4 <= 0

END

Model 4

<table>
<thead>
<tr>
<th>MAXIMIZE γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject To:</td>
</tr>
<tr>
<td>199200000-40000X1-33000X2-35000X3-32000X4 =&gt; 448100000</td>
</tr>
<tr>
<td>94.5 + 0.02X1 + 0.03X2 + 0.03X3 + 0.04X4 =&gt; 596.0</td>
</tr>
<tr>
<td>175.25 + 0.02X1 + 0.03X2 + 0.08X3 + 0.04X4 =&gt; 725.438</td>
</tr>
<tr>
<td>X1 + X2 + X3 + X4 = 12000</td>
</tr>
<tr>
<td>312.5 + X1 =&gt; 662.5</td>
</tr>
<tr>
<td>315.0 =&gt; 315</td>
</tr>
<tr>
<td>250.0 =&gt; 250</td>
</tr>
<tr>
<td>100 =&gt; 100</td>
</tr>
<tr>
<td>0.07X1+0.05X2+0.03X3+0.04X4 =&gt; 10920</td>
</tr>
<tr>
<td>0.04X1+0.03X2+0.08X3+0.04X4 =&gt; 600</td>
</tr>
<tr>
<td>125000000+40000X3 =&gt; 262500000</td>
</tr>
<tr>
<td>30000000+33000X2 =&gt; 150000000</td>
</tr>
<tr>
<td>100000000+35000X1 =&gt; 210000000</td>
</tr>
<tr>
<td>30000000+32000X4 =&gt; 630000000</td>
</tr>
</tbody>
</table>

Result analysis

Here the maximum degree of overall satisfaction (max) of 0.15 only is achieved for the solution S1 = 5268; S2 = 1404; S3 = 3374 and S4 = 1955. This provides the best solution at 5% uncertainty for an aggregate demand of 12000 tons, yielding Rs 437.7 millions as the net cost, 582 tons as total number of rejections and 699 tons as total number of late delivered items. In this fuzzy formulation, the quota allocations of suppliers 1 and 4 have increased by 41.6% and 123.4% as compared to the deterministic model, whereas the quota of vendor 2 and 3 has decreased by 53.2% and 23.4% as compared to the deterministic model. This is due to 5% vagueness that has been captured in the estimate of vendors’ capacities and budgets.

Results of application of models 3 & 4 to SQA problem with 5% to 50% vagueness created in vendor’s capacities and budgets

<table>
<thead>
<tr>
<th>% of Uncertainty</th>
<th>Degree of Satisfaction</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>Total Cost (Rs.)</th>
<th>Rejection</th>
<th>Late delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>0.131</td>
<td>5268</td>
<td>1404</td>
<td>3374</td>
<td>1955</td>
<td>437,606,902</td>
<td>381.8</td>
<td>699.3</td>
</tr>
<tr>
<td>10%</td>
<td>0.165</td>
<td>6773</td>
<td>3255</td>
<td>1976</td>
<td>0</td>
<td>447,365,480</td>
<td>494.3</td>
<td>625.0</td>
</tr>
<tr>
<td>15%</td>
<td>0.190</td>
<td>8708</td>
<td>3364</td>
<td>1297</td>
<td>0</td>
<td>486,210,440</td>
<td>460.7</td>
<td>609.6</td>
</tr>
<tr>
<td>20%</td>
<td>0.214</td>
<td>7332</td>
<td>3473</td>
<td>1297</td>
<td>0</td>
<td>449,217,300</td>
<td>487.2</td>
<td>595.0</td>
</tr>
<tr>
<td>25%</td>
<td>0.256</td>
<td>7444</td>
<td>3572</td>
<td>882</td>
<td>0</td>
<td>450,074,080</td>
<td>483.9</td>
<td>581.1</td>
</tr>
<tr>
<td>30%</td>
<td>0.310</td>
<td>8108</td>
<td>3892</td>
<td>0</td>
<td>0</td>
<td>452,756,770</td>
<td>475.5</td>
<td>557.7</td>
</tr>
<tr>
<td>35%</td>
<td>0.318</td>
<td>8108</td>
<td>3892</td>
<td>0</td>
<td>0</td>
<td>452,756,770</td>
<td>473.5</td>
<td>537.7</td>
</tr>
<tr>
<td>40%</td>
<td>0.219</td>
<td>8304</td>
<td>3801</td>
<td>0</td>
<td>0</td>
<td>454,124,954</td>
<td>481.8</td>
<td>540.9</td>
</tr>
<tr>
<td>45%</td>
<td>0.175</td>
<td>8571</td>
<td>3439</td>
<td>0</td>
<td>0</td>
<td>455,098,120</td>
<td>445.7</td>
<td>543.1</td>
</tr>
<tr>
<td>50%</td>
<td>0.165</td>
<td>8860</td>
<td>3448</td>
<td>0</td>
<td>0</td>
<td>458,527,250</td>
<td>428.4</td>
<td>549.8</td>
</tr>
</tbody>
</table>

A crisp-LPP formulation for case study SQA problem with 5% fuzziness about Vendor’s capacities and budget.
The above graphs clearly reveal that with the increase in degree of uncertainty / fuzziness about vendor’s deterministic parameters from 5% to 50%, the net cost of allocating 12000 tons to 4 vendors increases continuously while the rejections and late delivered items decrease with overall degree of satisfaction varying in a range of 0.15 to 0.25.

% change in quota allocations and objectives with 5% to 50% change in degree of uncertainty by FP.

<table>
<thead>
<tr>
<th></th>
<th>Cost</th>
<th>Rejection</th>
<th>Late Delivery</th>
<th>Degree of Satisf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>5268</td>
<td>1404</td>
<td>3374</td>
<td>1955</td>
</tr>
<tr>
<td>10%</td>
<td>6773</td>
<td>3251</td>
<td>1976</td>
<td>0</td>
</tr>
<tr>
<td>15%</td>
<td>7069</td>
<td>3364</td>
<td>1627</td>
<td>0</td>
</tr>
<tr>
<td>20%</td>
<td>7332</td>
<td>3471</td>
<td>1297</td>
<td>0</td>
</tr>
<tr>
<td>25%</td>
<td>7444</td>
<td>3573</td>
<td>982</td>
<td>0</td>
</tr>
<tr>
<td>30%</td>
<td>8108</td>
<td>3892</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>35%</td>
<td>8108</td>
<td>3892</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>40%</td>
<td>8304</td>
<td>3696</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>45%</td>
<td>8571</td>
<td>3429</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50%</td>
<td>8860</td>
<td>3140</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In fuzzy programming model, an attempt is made to determine the vendor’s quota in a supply chain when various parameters of vendors are not
known with certainty. The advantages of the fuzzy modeling is that the complexity of the vendor quota allocation problem may be handled even if the capacity of each vendor is vague, which may be due to limited sharing of internal data between buyer and supplier.

Suggestions / recommendation
1. S1 is the most preferred supplier in the approved vendor’s list of the firm because as compared to other three vendors his:
   ✓ Vendor rating of 97% is the highest.
   ✓ Quality is the best (as % rejection is only 0.02%).
   ✓ Capacity to supply is maximum.
   ✓ Budget to supply is enormous.

2. S4 has quoted the lowest price of Rs 32000 per ton but his vendor rating is very poor and he has low capacity to supply. The most negative aspect of S4 is the high rejection rate of 0.10%, which may perhaps be the reason for his low pricing.

3. S2 in his favour has best performance in on-time delivery of items to the company, which is commendable. Most of his other performance criteria are always of second degree in comparison to other vendors.
   ✓ His quoted price of Rs. 33000 per ton is only one thousand rupees more than S4.
   ✓ His flexibility in supply of items is also second best after S4.
   ✓ The vendor rating of 0.90% of S2 is second best only after S1.
   ✓ But his poor quality of product (with rejection level of 0.08%) is only better than S4— again second from bottom. Also this poor quality is a major contributor in lowering his vendor rating.

4. S3 has none of the performance criteria in his favour. He is either moderately good or moderately poor.
   ✓ In slightly favourable position S3 has rejection level of 0.05% which makes him better placed after S1 in supplying good quality material. Also S3 has second highest capacity to supply and budget allocation after S1.

Finally based on the study, the preference order of selection of vendors that could be considered for optimal quota allocation is: S1 > S2 > S3 > S4.

References