In this paper, a path generation problem has been solved by carrying out approximate dimensional synthesis of six-bar, single degree of freedom Watt-II mechanism. The mechanism consisting of all revolute pairs is capable to trace the prescribed trajectory which comprises of twenty four precision points. The dimensional synthesis is carried out using analytical complex number method wherein, the standard dyad/triad and loop closure equations are generated. The final lengths and orientations of various links of the mechanism are determined by solving these equations using a MATLAB code for twenty four coupler displacement positions. The dimensional synthesis of the mechanism is demonstrated and verified on a numerical example.

**KEYWORDS:** Dimensional Synthesis, Path Generation, Precision Points, Watt-II Mechanism, Dyad/Triad Equations.

**I. INTRODUCTION**

The Kinematic synthesis deals with the methods of creating useful mechanisms having desired motion performance. It involves various steps e.g., type synthesis, number synthesis and dimensional synthesis. Dimensional synthesis is the final step in Kinematic synthesis and involves determination of various unknown dimensions of a predefined mechanism. It deals with determination of major dimensions and home positions of all links with their orientations of the kinematic linkages. The linkage may be a slider crank mechanism, a cam-follower, a four-bar mechanism or any other linkage with higher number of links. The task for which the kinematic linkage is synthesized is path generation, motion generation, rigid body guidance and function generation etc. The major dimensions of the kinematic linkage include link lengths (i.e. joint-to-joint distances, diameters of cam-follower, cam-contour dimensions and their eccentricities etc. Many researches carried out the dimensional synthesis of various kinematic linkages using graphical and analytical methods for different kinematic tasks. The analytical methods are simpler and have advantages of improved accuracy, ease of generality whereas graphical methods are confined to drawing accuracy only.

Based on graphical method, the dimensional synthesis of five-bar mechanism was performed by Rose and Rawat [1-2]. An adjustable four bar mechanism was synthesized by Naik and Amarnath [3] using five bar loop closure equations based on function generators. The variable topology five-bar mechanism synthesis was projected by Balli and Chand [4-5] for motion between extreme positions. They employed complex number mathematics in their work. Later, the five-bar mechanism synthesis work was extended to mechanism with two binary offset tracing links [6-7]. Also, the variable topology synthesis of seven-bar mechanisms was suggested by various researchers [8-11]. They suggested the syntheses of these mechanisms for motion between extreme positions based on kinematic tasks viz., path generation, motion generation and function generation etc. The syntheses of variable topology five-bar and seven-bar slider mechanisms were jointly performed by Daivagna and Balli [12-14] for finitely separated positions.

Researchers projected synthesis of six-bar, single degree of freedom mechanisms based on limited number of precision points path generation. A six-bar mechanism was dimensionally synthesized for 8 precision points based on path generation [15]. Later, the path generation dimensional synthesis of six-bar Stephenson-II and...
Stephenson-III mechanisms was carried out for 15 and 12 precision points respectively [16-17]. In automation industry, to trace the complicated paths for a given mechanism, more precision points are needed. Therefore, in present work, a six-bar Watt II mechanism is considered for dimensional synthesis based on path generation. The trajectory of the desired path is defined by twenty four precision points. The standard dyad/triad technique has been adopted for generating required analytical loop closure equations using complex number mathematics. The dimensional synthesis work of Watt-II mechanism has been demonstrated on a number problem. Finally, the solution has been obtained and verified by coding with MATLAB-R2012a.

II. CONFIGURATION OF SIX-BAR WATT-II MECHANISM

A Watt-II is a six-bar mechanism. It consists of two ternary links and four binary links. In Watt-II mechanism, the fixed link is a ternary link. The other ternary link directly forms a revolute pair at one end of the fixed ternary link while the other ends of the fixed ternary link are connected to binary links. One binary link of the Watt-II mechanism has an offset.

The configuration of Six-bar Watt-II mechanism is shown in Fig. 1. The link 1 is ternary link which is fixed at pivots O₁, O₂ and O₃. The link 2 is a binary crank link which rotates about pivot point O₁. The input motion is supplied to the mechanism through binary crank link 2. The link 3 is a binary link with offset at point B, which is connected with other links at point A and C. The link 4 is a ternary link, which is connected with two binary links and one ternary link at joints C, D and O₂ respectively. Also, the point C is the tracing point of the mechanism for which it is dimensionally synthesized. The link 5 and link 6 are the two binary links connected at common joint E. The other ends of link 5 and link 6 are connected to link 4 and link 1 at joints D and E respectively.

III. GENERATION OF LOOP CLOSURE EQUATIONS

Consider the initial arrangement of the given Six-bar Watt-II Mechanism expressed by joint positions O₁A₀B₀C₀ O₂D₀E₀O₃. When the crank O₁A₀ rotates through angle θ and reaches position O₁Aₗ, the joint positions of
The displacement of tracing point from position $B_0$ to $B_j$ is expressed by $\delta_j$. The derivation of loop closure equations for 24 precision points is explained below:

Writing the loop closure equation [18] for independent vector loop $O_1A_jB_jB_0A_0$ (Refer Fig. 2)

$$Z_1(e^{i\theta_j}) + Z_3(e^{i\alpha_j}) - \delta_j - Z_5 - Z_7 = 0$$

(1)

Writing the loop closure equation [18] for independent vector loop $O_2C_jB_jB_0C_0$ (Refer Fig. 2)

$$Z_2(e^{i\beta_j}) + Z_3(e^{i\beta_j}) - \delta_j - Z_5 - Z_7 = 0$$

(2)

Writing the loop closure equation [18] for independent vector loop $O_2E_jD_jB_jB_0D_0E_0$ (Refer Fig. 2)

$$Z_2(e^{i\theta_j}) + Z_3(e^{i\alpha_j}) + Z_5(e^{i\beta_j}) + Z_7(e^{i\gamma_j}) - \delta_j - Z_5 - Z_7 = 0$$

(3)

In equations (1) to (3), value of $j$ varies from 1 to 24 for twenty four points comprising the required trajectory.

Considering closed loop $C_0A_0B_0C_0$, we get unknown vector $Z_2$ (Refer Fig. 2)

$$Z_2 = Z_4 - Z_3$$

(4)

Considering closed loop $O_2D_0C_0O_2$, we get unknown vector $Z_5$ (Refer Fig. 2)

$$Z_5 = Z_5 - Z_6$$

(5)

Considering closed loop $O_2E_0D_0O_2$, we get unknown vector $Z_{11}$ (Refer Fig. 2)

$$Z_{11} = Z_5 - Z_6$$

(6)

Considering closed loop $O_1O_2C_0A_0O_1$, we get unknown vector $Z_{10}$ (Refer Fig. 2)

$$Z_{10} = Z_1 - Z_7$$

(7)
IV. NUMERICAL PROBLEM BASED ON SYNTHESIS OF SIX-BAR WATT-II MECHANISM

Problem Statement:
It is required to synthesize a Six-bar Watt-II mechanism which transmits motion along a path prescribed by twenty four precision points (Refer Fig. 3) \( P_1 (0.344, 0.527), P_2 (0.284, 0.523), P_3 (0.224, 0.508), P_4 (0.169, 0.483), P_5 (0.121, 0.449), P_6 (0.082, 0.408), P_7 (0.054, 0.365), P_8 (0.037, 0.321), P_9 (0.031, 0.280), P_{10} (0.036, 0.244), P_{11} (0.049, 0.216), P_{12} (0.071, 0.197), P_{13} (0.098, 0.190), P_{14} (0.132, 0.195), P_{15} (0.170, 0.215), P_{16} (0.214, 0.249), P_{17} (0.269, 0.295), P_{18} (0.335, 0.349), P_{19} (0.406, 0.395), P_{20} (0.462, 0.429), P_{21} (0.485, 0.456), P_{22} (0.477, 0.481), P_{23} (0.446, 0.503) \) and \( P_{24} (0.400, 0.520) \).

Prescribed Parameters:
The prescribed parameters are displacement of each point from initial position i.e. \( \delta_j = P_j - P_1 \) at equal intervals of \( \theta_j \{0, 2\pi\} \) (where \( j = 1, 2...24 \)).

Assumed Parameters:
The parameters assumed freely are \( a, b, c, \) and \( d \). The range of these parameters are \( a_j \{0, \pi/3\}, b_j \{-\pi/12, \pi/3\}, c_j \{-\pi/8, 5\pi/12\} \) and \( d_j \{-\pi/24, \pi/16\} \) [19].

Design Parameters:
The MATLAB code is developed to solve for the design vectors \( Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_9 \) and \( Z_{10} \).

V. SOLUTION OF LOOP CLOSURE EQUATIONS
The solution of equations (1) to (7) is difficult to find manually for twenty four precision points. Also, the number of equations is more than the number of unknowns, so a code is developed in MATLAB to solve these equations. To obtain solution, MATLAB algorithm consists of following steps:

Step 1. Read the value of twenty four precision points coordinates.

Step 2. Calculate displacement (\( \delta_j \)) for each point by subtracting its coordinates from initial point.
Step 3. Read the values of all free parameters and assumed parameters.

Step 4. Calculate values $e^{i\theta_j}$, $e^{i\alpha_j}$, $e^{i\beta_j}$, $e^{i\gamma_j}$ and $e^{i\phi_j}$ for $j = 1, 2, 3, \ldots, 24$.

Step 5. Calculate design vectors $Z_1$ and $Z_1$ using eq. (1).

Step 6. Calculate design vectors $Z_7$ and $Z_4$ using eq. (2).

Step 7. Calculate design vectors $Z_4$, $Z_9$ and $Z_6$ using eq. (3).

Step 8. Calculate design vector $Z_1$ using eq. (4).

Step 9. Calculate design vector $Z_1$ using eq. (5).

Step 10. Calculate design vector $Z_{11}$ using eq. (6).

Step 11. Calculate design vector $Z_{10}$ using eq. (7).

VI. RESULTS AND DISCUSSION

The dimensions and orientations of each link obtained by solving loop closure equations using MATLAB code are:

$Z_1 = 0.223 \text{ m} \angle 88.35^\circ$; $Z_2 = 0.587 \text{ m} \angle 210.88^\circ$; $Z_3 = 0.313 \text{ m} \angle 40.57^\circ$; $Z_4 = 0.283 \text{ m} \angle 200.19^\circ$; $Z_5 = 0.281 \text{ m} \angle 80.33^\circ$; $Z_6 = 0.249 \text{ m} \angle 96.37^\circ$; $Z_7 = 0.525 \text{ m} \angle 87.83^\circ$; $Z_8 = 0.173 \text{ m} \angle 56.15^\circ$; $Z_9 = 0.286 \text{ m} \angle 152.12^\circ$; $Z_{10} = 0.490 \text{ m} \angle 0.01^\circ$; $Z_{11} = 0.203 \text{ m} \angle 0.05^\circ$.

The final dimensions of each link of Six-bar Watt-II mechanism based on twenty four precision points path generation are graphically shown in Fig. 4.
VII. CONCLUSION

The present work suggests path generation dimensional synthesis of a Six-bar Watt-II mechanism which will help the automation industry to control the sophisticated industrial machining processes. The mechanism transmits motion for a prescribed trajectory that comprises of twenty four precision points. The present work will facilitate in inventing useful mechanisms where tracing point is to be dedicated along a complex path to carry out industrial operations. The results of a numerical example have been obtained with the help of MATLAB code without much iteration. The solved numerical example signifies the effectiveness of the present work. This technique has the advantages of reduced solution space with improved accuracy over graphical methods. The present work is further extendable for various other kinematic tasks of rigid body guidance and function generation etc.

VIII. NOMENCLATURE

\[ Z_i \] : Length of each side of all of links (i = 1, 2, 3,...11)
\[ \delta_j \] : Displacement of tracing point from position B_0 to B_j (j = 1, 2, 3...24)
\[ \theta_i \] : Angle through which crank O_A0 rotates and reaches at position O_A1
\[ \alpha_j \] : Angle through which binary offset link A_B0C_0 rotates and reaches at position A_B1C_1
\[ \beta_i \] : Angle through which ternary link O_CD0 rotates and reaches at position O_CD1
\[ \gamma_j \] : Angle through which binary link O_CE0 rotates and reaches at position O_CE1
\[ \phi_j \] : Angle through which binary link D_CE0 rotates and reaches at position D_CE1

IX. REFERENCES

Appendix

The loop closure equation (1) for Six bar Watt-II mechanism is

\[ Z_i(e^{\theta_i} - 1) + Z_j(e^{\theta_j} - 1) = \delta_j \quad \text{for twenty four precision points, say (j = 1, 2, \ldots, 24)}; \]

The above equation can be formulated in the form as

\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_1 \quad \text{(8)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_2 \quad \text{(9)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_3 \quad \text{(10)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_4 \quad \text{(11)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_5 \quad \text{(12)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_6 \quad \text{(13)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_7 \quad \text{(14)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_8 \quad \text{(15)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_9 \quad \text{(16)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{10} \quad \text{(17)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{11} \quad \text{(18)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{12} \quad \text{(19)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{13} \quad \text{(20)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{14} \quad \text{(21)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{15} \quad \text{(22)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{16} \quad \text{(23)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{17} \quad \text{(24)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{18} \quad \text{(25)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{19} \quad \text{(26)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{20} \quad \text{(27)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{21} \quad \text{(28)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{22} \quad \text{(29)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{23} \quad \text{(30)} \]
\[ Z_1(e^{\theta_1} - 1) + Z_1(e^{\theta_1} - 1) = \delta_{24} \quad \text{(31)} \]

The loop closure equation (2) for Six bar Watt-II mechanism is

\[ Z_1(e^{\theta_1} - 1) + Z_2(e^{\theta_2} - 1) = \delta_j \quad \text{for twenty four precision points, say (j = 1, 2, \ldots, 24)}; \]

The above equation can be formulated in the form as

\[ Z_1(e^{\theta_1} - 1) + Z_2(e^{\theta_2} - 1) = \delta_1 \quad \text{(32)} \]
The loop closure equation (2) for Six bar Watt- II mechanism is

\[ Z_j(e^{\beta_j} - 1) + Z_k(e^{\alpha_k} - 1) = \delta_j \]  

for twenty four precision points, say \( j = 1, 2, \ldots, 24 \);

The above equation can be formulated in the form as

\[ Z_j(e^{\beta_j} - 1) + Z_k(e^{\phi_k} - 1) + Z_l(e^{\beta_l} - 1) + Z_m(e^{\alpha_m} - 1) = \delta_j \]  

The loop closure equation (2) for Six bar Watt- II mechanism
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{12} \]  
\[ Z_4(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{13} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{14} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{15} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{16} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{17} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{18} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{19} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{20} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{21} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{22} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{23} \]  
\[ Z_6(e^{y_{12}} - 1) + Z_8(e^{y_{13}} - 1) + Z_4(e^{y_{14}} - 1) + Z_4(e^{y_{15}} - 1) = \delta_{24} \]  

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