ABSTRACT

Analysis has been conducted to analyze the effects of second order slip flow and heat transfer of Jeffrey nanofluid over a stretching sheet with non linear thermal radiation and chemical reaction. The effects of Brownian motion and thermophoresis occur in the transport equations. The velocity, temperature and nanoparticle concentration profiles are analyzed with respect to the involved parameters of interest namely Brownian motion parameters, thermophoresis parameter, magnetic parameter, radiation parameter, Prandtl number, Lewis number, chemical reaction parameter, and Deborah number, Convergence of the derived solutions was checked and the influence of embedded parameters was analyzed by plotting graphs. It was noticed that the velocity increases with an increase in the Deborah number. We further found that for fixed values of other parameters, numerical values of the skin friction coefficient, local Nusselt numbers and Sherwood numbers were computed and examined. A comparative study between the previous published and present results in a limiting sense is found in an excellent agreement.

KEYWORDS: Jeffrey fluid, nonlinear thermal radiation, chemical reaction, second order slip, numerical solution.

INTRODUCTION

The boundary layer flow analysis of an electrically conducting fluid due to a stretching sheet is of great interest because of their diverse engineering and industrial applications. MHD has immediate applications in designing of heat exchangers, in space vehicle propulsion, in thermal protection, inagnetohydrodynamic (MHD) power generators, MHD pumps, in polymer technology, in petroleum industry, in purification of crude oil and fluid droplets sprays. Its relevance is also seen in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In this view, many authors [1–6] have recently studied the MHD effects on flow problems with different aspects. They found that, MHD effect have a significant role in thermal management applications.

In the recent years, micro-scale fluid dynamics in the Micro-Electro-Mechanical Systems (MEMS) received much attention in research. Because of the micro-scale dimensions, the fluid flow behavior belongs to the slip flow regime and greatly differs from the traditional flow [7]. For the flow in the slip regime, the fluid motion still obeys the Navier–Stokes equations, but with slip velocity or temperature boundary conditions. In addition, partial velocity slips over a moving surface occur for fluids with particulate such as emulsions, suspensions, foams, and polymer solutions [8]. The slip flows under different flow configurations have been studied in the literature [9–14]. Hayat et al. [15] studied the steady three-dimensional boundary layer flow of water based nanofluid with copper as nanoparticle over a permeable stretching surface with second order velocity slip and homogeneous–heterogeneous reactions. Zhu et al. [16] are investigated the effects of the second-order velocity slip and temperature jump boundary conditions on the magnetohydrodynamic (MHD) flow and heat transfer of water-based nanofluids containing Cu and Al₂O₃ in presence of thermal radiation. Megahed [17] obtained numerical solution to study the boundary layer flow and heat transfer for an electrically conducting Casson fluid over a permeable stretching surface with second-order slip velocity model and thermal slip conditions in the presence of internal heat generation/absorption and thermal radiation and he shown that increasing the velocity and thermal slip parameters makes the rate of heat transfer decrease. Hakeem et al. [18] performed both numerical and analytical analysis to study the effect of magnetic field on a steady two dimensional laminar radiative flow of an
incompressible viscous water based nanofluid over a stretching/shrinking sheet with second order slip boundary condition.

Besides, the radiative heat transfer have wide occurrence in various applications, such as in nuclear power plants, gas turbines, propulsion devices for space vehicles, missiles and aircraft etc. In view of these applications, many researchers [19–23] have considered the influence of thermal radiation effect with different physical situations. To simplify the radiative heat flux the Rosseland approximation has been employed. Further, they have assumed small temperature differences within the flow to make out the linear radiative heat flux. But in recent years, many authors have an interest in the study of non-linear thermal radiation effect (see [24–27]).

It is now a well-accepted fact that many fluids of industrial and geophysical importance are non-Newtonian. Due to much attention in many industrial applications, such as the extrusion of plastic sheets, fabrication of adhesive tapes, glass-fiber production, metal spinning, and drawing of paper films. Recently, some research has been focused on the study of nanofluids. Nanofluids are a homogenous mixture of a base fluid and nanoparticles. The term nanofluid was first introduced by Choi [28] to describe engineered colloids composed of nanoparticles dispersed in a base fluid. Many studies are focused on non-Newtonian fluid as a base fluid with suspended nanoparticles over a stretching sheet [29, 30, 31] Hayat et al. [32] studied the effects of thermophoresis and Brownian motion on the three-dimensional (3D) boundary layer flow and convective heat transfer of Jeffrey nanofluid over a bidirectional stretching surface with newly developed boundary condition with the zero nanoparticles mass flux. Shehzad et al. [33, 34] investigated the effects of convective heat and concentration conditions in magnetohydrodynamic two-dimensional and three-dimensional flow of Jeffrey nanofluid fluid with nanoparticles. Dalira et al. [35] numerically studied the entropy generation for steady laminar two-dimensional forced convection magnetohydrodynamic (MHD) boundary layer flow, heat transfer and mass transfer of an incompressible non-Newtonian nanofluid over a linearly stretching, impermeable and isothermal sheet with viscous dissipation. Recently, Prasannakumara et al [36] studied the Effects of chemical reaction and nonlinear thermal radiation on Williamson nanofluid slip flow over a stretching sheet embedded in a porous medium.

The purpose of present paper is to analyze the effect of second order slip and nonlinear thermal radiation on heat and momentum transfer of steady two-dimensional slip flow of a nanofluid over a stretching sheet. Reduced governing nonlinear ordinary differential equations are solved numerically by means of Runge-Kutta-Fehlberg-45 order method. The effects of different flow parameters on flow fields are elucidated through graphs and tables.

MATHEMATICAL FORMULATION

Let us consider a steady flow of an incompressible Jeffrey nanofluid over a horizontal stretching surface. The flow region is confined to $y > 0$ and the plate is stretched along $x$-axis with a velocity $U_w = ax$, where $a$ is a positive constant. A uniform magnetic field $B_0$ is applied in the transverse direction $y$ normal to the plate. The nanofluid is assumed to be single phase, in thermal equilibrium and there is a slip velocity between the base fluid and particles. The stretching surface temperature and the nanoparticles fraction are deemed to have constant value $T_w$ and $C_w$, respectively. The ambient fluid temperature and nanoparticles fraction have constant value $T_m$ and $C_m$, respectively. The coordinate system and flow regime is illustrated as shown in the figure 1.

It is well known that the constitutive equations for a Jeffrey fluid are given by (2015)

$$
\tau = -p I + S, \\
S = \frac{\mu}{1 + \lambda} \left[ R_1 + \lambda_1 \left( \frac{\partial u}{\partial t} + V \cdot \nabla \right) R_1 \right],
$$

where $\tau$ is the Cauchy stress tensor, $S$ is the extra stress tensor, $\mu$ is the dynamic viscosity, $\lambda$ and $\lambda_1$ are the material parameters of Jeffrey fluid and $R_1$ is the Rivlin–Ericksen tensor defined by

$$
R_1 = (VV) + (VV)',
$$

Under usual boundary layer approximations governing two-dimensional equations for the present problem are given as follows (2009):

$$
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\nu}{1 + \lambda} \left[ \frac{\partial^2 u}{\partial y^2} + \lambda_1 \left( \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x \partial y} \right) \right] = \frac{\sigma h_0^2}{\rho_f} u,
$$

$$
(2.1)
$$

$$
(2.2)
$$

The governing equations can be reduced to ordinary differential equations, using the following similarity transformations,

\[
\begin{align*}
    u &= ax' \phi(\eta), \\
    v &= -\sqrt{aw} f(\eta), \\
    T &= T_\infty \left(1 + (\theta_w - 1)\theta(\eta)\right), \\
    \phi(\eta) &= \frac{C - C_\infty}{C_\infty - C_\infty}.
\end{align*}
\] (10)

Where \( \theta_w = T_w / T_\infty \) and \( \theta_w > 1 \) the temperature ratio parameter (Shehzad et al. 2014).

With the help of aforementioned transformations, equation (2.1) is identically satisfied and equations (2.2), (2.4) and (2.9) will take the following forms:

\[
\begin{align*}
    f'''' + (1 + \lambda)[f''' - f'''] + \beta[f'''' - f f'''] - (1 + \lambda)(M) f' &= 0, \\
    [1 + Nr \left(1 + (\theta_w - 1)\theta\right)] f' + Pr[f \theta' + Nb \phi' \theta' + Nt(\theta')^2] &= 0, \\
    \phi'' + Le \phi' + \frac{Nt}{N_b} \phi'' - \gamma \phi &= 0.
\end{align*}
\] (11-13)
The corresponding boundary conditions are:

\[ f(0) = 0, \quad f'(0) = 1 + A_1 f'''(0) + A_2 f''''(0), \quad \theta(0) = 1, \quad \phi(0) = 1 \quad \text{at} \ \eta = 0, \]

\[ f'(\eta) = f'''(\eta) = \theta(\eta) = \phi(\eta) = 0 \quad \text{as} \ \eta \to 0, \quad (2.14) \]

where \( f, \theta \) and \( \phi \) are functions of \( \eta \) and prime denotes derivatives with respect to \( \eta \). \( \beta = \alpha A_1 \) is Deborah number, \( M = \frac{\sigma B_0^2}{\rho f_a} \) is magnetic parameter called Hartmann number, \( N_r = \frac{1 - 6A^2 \gamma^3}{3k \kappa} \) is radiation parameter, \( N_b = \frac{\tau_0 (\gamma - C_0)}{v} \) is Brownian motion parameter, \( N_t = \frac{\tau_0 (\gamma - C_∞)}{v T_∞} \) is thermophoresis parameter, \( Pr = \frac{v}{\nu} \) is Prandtl number, \( \gamma = \frac{k_a L_e}{a} \) is chemical reaction parameter, and \( Le = \frac{v}{a} \) is Lewis number.

The skin friction coefficient \( (C_{f_x}) \), local Nusselt number \( (Nu_x) \) and Local Sherwood number \( (Sh_x) \) are given by,

\[
C_{f_x} = \frac{\tau_w}{\rho u_0^2}, \quad Nu_x = \frac{x q_w}{k (T_w - T_0)} \quad \text{and} \quad Sh_x = \frac{x q_m}{k (C_w - C_0)}, \quad (2.15)
\]

where the shear stress along the stretching surface \( \tau_w \), the surface heat flux \( q_w \) and the surface mass flux \( q_m \) are

\[
\tau_w = \frac{k}{1 + \lambda} \left[ A_1 \left( \frac{\partial u}{\partial y} \right) + A_2 \left( \frac{\partial^2 u}{\partial x \partial y} + u \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial y^2} \right) \right] \bigg|_{y = 0},
\]

\[
q_w = -k \frac{\partial T}{\partial y} + (q_r)_w, \quad q_m = -D_b \frac{\partial C}{\partial y} \quad \text{at} \quad y = 0. \quad (2.16)
\]

Substituting the values of \( \tau_w, q_w \) and \( q_m \) into the equation \( (2.16) \) we have

\[
\sqrt{Re} C_{f_x} = \frac{1}{\sqrt{Re}} \left( f''(0) + \beta f'(0) f'''(0) - f'(0)f''(0) \right).
\]

\[
\frac{Nu_x}{\sqrt{Re}} = -\left( 1 + N_r \gamma^3 \right) u'(0), \quad \frac{Sh_x}{\sqrt{Re}} = -\phi'(0), \quad (2.17)
\]

where \( Re = \frac{ax^2}{v} \) is local Reynolds number.

NUMERICAL METHOD

The system of non-linear ordinary differential equations \((2.11)\) to \((2.13)\) with boundary conditions \((2.14)\) have been solved using Runge-Kutta-Fehlberg fourth-fifth order method along with Shooting technique. The method has the following steps: In the first step, the governing system of Eqs. \((2.11)\) to \((2.13)\) are reduced to a system of eight simultaneous differential equations of first order by introducing new dependent variables. After fixing finite value for \( \eta_0 \), integration is carried out with the help of Runge-Kutta-Fehlberg-45 (RKF-45) method. Runge-Kutta-Fehlberg-45 method has a procedure to determine if the proper step size \( h \) is being used. At each step, two different approximations for the solution are made and compared. If the two answers are in close agreement, the approximation is accepted otherwise, the step size is reduced until to get the required accuracy. For the present problem, we took step size \( \Delta \eta = 0.001 \), \( \eta_0 = 5 \) and accuracy to the fifth decimal places. To have a check on the accuracy of the numerical procedure used, first test computations for \( \theta'(0) \) are carried out for viscous fluid for various values of \( Pr \) and compared with the available published results of Goyal and Bhargava (2014), Gorla and Sidawi (1994), Nadeem and Hussain (2013) and Wang (1989) in table 1 and they are found to be in excellent agreement.

RESULTS AND DISCUSSION

A theoretical investigation of Second order velocity slip boundary layer flow of Jeffrey nanofluid over a stretching sheet under the influence of nonlinear thermal radiation and chemical reaction has been performed. The value of the Prandtl number for the base fluid is kept as \( Pr = 10 \). The default values of the other parameters are mentioned in the description of the respected figures. In order to study the characteristics of velocity and temperature distribution for first order velocity slip parameter \( (A_1) \) and second order velocity slip parameter \( (A_2) \), Radiation parameter \( (N_r) \), temperature ratio parameter \( (\theta_0) \), magnetic parameter \( (M) \) graphs are plotted and physical reasons behind the trend of the graphs are discussed.

Effect of first order and second order velocity slip parameters on velocity and temperature profiles are demonstrated in Fig.2 and 3. We can observe that the effects of increasing values of both first and second order velocity slip parameters are reduces the thickness of momentum boundary layer and hence decrease the velocity. Therefore, increasing values of velocity slip coefficients \( (A_1 \text{ and } A_2) \) decrease the boundary layer velocity, where
Fig. 4 describes the effects of Deborah number $\beta$ on the velocity and temperature profiles. We can see that the boundary layer thickness and the fluid velocity increases with increase in $\beta$. It is because, increase in $\beta$ decreases the resistance of fluid motion which thus causes a higher fluid movement at the neighborhood of the stretching surface. Fig. 4 also reveals that larger values of Deborah number leads to a reduction in the temperature and thermal boundary layer thickness. It is due to the fact that Deborah number is directly proportional to relaxation time and larger values of Deborah number corresponds to the higher relaxation time. Such increase in relaxation time corresponds to the lower temperature and weaker thermal boundary layer thickness. We can also see that boost in $\beta$ causes the reduction in the concentration boundary layer.

Influence of $\lambda$ on velocity and temperature profile is highlighted in Fig. 5. It can be seen that increase in $\lambda$ decreases the fluid velocity but enhances temperature profile and it gives rise to the nanoparticle concentration field and associated boundary layer thickness. It is due to the fact that increase of $\lambda$ corresponds to a decrease in retardation time but increase in the relaxation time and hence higher values of $\lambda$ imply the domination of relaxation time over retardation time due to which temperature concentration profile are enhanced.

Fig. 6 shows the effect of magnetic parameter $M$ on dimensionless velocity and temperature distributions, respectively. The presence of a magnetic field in an electrically conducting fluid induces a force called Lorentz force, which opposes the flow. This resistive force tends to slow down the flow, so the effect of $M$ decreases the velocity and also cause increase in its temperature distributions.

Figs. 7 and 8 illustrates the effect of temperature ratio parameter $\theta_w$ on temperature profiles, when $Pr = 6.2$ and $Pr = 10$ respectively. From these plots, one can notice that, the increase in temperature ratio parameter increases the thermal state of the fluid, and it results in increase of temperature profiles. The effect of radiation parameter on temperature is depicted as in Fig. 9. A critical observation shows that, the temperature profile increases with increase in $Nr$. This is because, an increase in the radiation parameter provides more heat to fluid that causes an enhancement in the temperature and thermal boundary layer thickness.

Effect of chemical reaction parameter $\gamma$ on nanoparticle volume fraction profile is shown in Fig. 10 for the several values of $\gamma(>0)$ and $\gamma(<0)$ cases. It is observed that the nanoparticles volume fraction decreases for constructive chemical reaction parameter and increases for destructive chemical reaction parameter.

Figs. 11 and 12 displays the effect of Lewis number $Le$ on temperature and concentration profiles. From these figures both the profiles decreases with increasing the values of the $Le$. It is due to the fact that the larger values of Lewis number makes the mass diffusivity smaller, therefore it decreases the concentration field.

Temperature and nanoparticles volume fraction variation against different values of $Nb$ and $Nt$ are depicted respectively, as in Figs. 13 to 15. We can see that the temperature profiles are increasing function of $Nb$, whereas nanoparticles volume fraction is a decreasing. This may be due to the fact that as a Brownian motion parameter $Nb$ decreases the mass transfer of a nanofluid. Further both temperature and nanoparticles volume fraction profiles increases for increasing values of $Nt$. The variation in Prandtl number Pron $\theta$ is shown in Fig. 16. The temperature field $\theta$ decreases when $Pr$ increases. It is obvious that an increase in the values of $Pr$ reduces the thermal diffusivity therefore thermal boundary layer thickness is decreasing function of $Pr$.

The numerical results are recorded in table 2, and it illustrates the variation of skin friction co-efficient and Nusselt number with respect to various flow controlling parameters. As expected, both first order and second order velocity slip parameters effect is to reduce the friction at the solid-fluid interface, and thus reduces the skin friction coefficient. Similar behaviour is also observed in the case of $\lambda$, i.e in the presence velocity slip, increase in $\lambda$ results decrease of both skin friction coefficient and local Nusselt number. But quite opposite behaviour is observed in case of $\beta$ and $M$.

The effects of various pertinent parameters on local Nusselt number and local Sherwood number is discussed numerically through table 3. We can see that $\gamma, Le$ and $Pr$ shows favourable effect on coefficient of $\phi'(0)$, whereas effect of $\theta_w, Nb$ and $Nt$ on local Nusselt number is negligible. We can also observe that both $\theta_w$ and
Pr show positive effect on local Nusselt number. This is due to the fact that a higher Prandtl number reduces the thermal boundary layer thickness and increases the surface heat transfer rate. Also high Prandtl number implies more viscous fluid which tends to retard the motion. Similarly and \( \theta_w \) shows negative effect and chemical parameter has no effect on local Nusselt number.

**CONCLUSION**

A boundary layer analysis to study the effect of nonlinear thermal radiation on Second order Slip flow and heat transfer of Jeffrey nanofluid over a stretching sheet with chemical reaction is presented. Numerical results for velocity profiles, surface heat transfer rate and mass transfer rate have been obtained for parametric variations of various ranges of slip boundary condition and for different values of flow pertinent parameters. The main outcomes of the problem are summarized as follow:

- Both first and second order velocity slip parameters are reduces the thickness of momentum boundary layer and hence decrease the velocity.
- Boundary layer thickness and the fluid velocity increases with increase in Deborah number.
- An increase in Lewis and Prandtl numbers shows a decrease in nanoparticles concentration.
- Larger values of magnetic parameter \( M \) lead to an enhancement in the temperature and nanoparticles concentration.
- Increase in \( \lambda \) and \( \theta_w \) enhances the temperature profile.
- Nanoparticles volume fraction decreases for constructive chemical reaction parameter and increases for destructive chemical reaction parameter.
- Both temperature and nanoparticles volume fraction increase for increasing values of \( Nr \).
- \( Nr \) enhances coefficient of Nusselt number, but the parameters \( \theta_w, Nb, Nt \) decreases – \( \theta'(0) \).

**NOMENCLATURE**

| \( u, v \) | velocity components along the x and y axes |
| \( k^* \) | Rosseland mean absorption coefficient |
| \( D_B \) | Brownian diffusion coefficient |
| \( D_T \) | thermophoresis diffusion coefficient |
| \( \beta_0 \) | magnetic field strength |
| \( c_f \) | local skin friction coefficient |
| \( Nu_x \) | local Nusselt number |
| \( \phi h_x \) | local Sherwood number |
| \( k_1 \) | chemical reaction coefficient |
| \( A_1 \) | is the first-order velocity slip parameter |
| \( A_2 \) | is the second-order velocity slip parameter |
| \( C \) | volumetric volume expansion coefficient |
| \( T \) | temperature of the nanofluid near wall |
| \( T_{ws} \) | fluid temperature far away from the sheet |
| \( T_0 \) | uniform wall temperature |
| \( k \) | thermal conductivity |
| \( U_w \) | stretching velocity |
| \( \alpha \) | stretching rate |
| \( M \) | magnetic parameter |
| \( Re_v \) | local Reynolds number |
| \( Pr \) | Prandtl number |
| \( Nr \) | radiation parameter |
| \( Le \) | Lewis number |
| \( Nb \) | Brownian motion parameter |
| \( Nr \) | thermophoresis parameter |

**Greek symbols**

- \( \lambda, A_1 \) ratio of relaxation and retardation times and the relaxation time
- \( \rho_f \) density of the fluid
- \( \rho_p \) nanoparticles density
- \( \theta \) dimensionless temperature variable
- \( \phi \) nanoparticles volume fraction
- \( \sigma \) thermal diffusivity
- \( \eta \) similarity variable
- \( \nu \) kinematic viscosity
- \( \sigma^* \) Stefan-Boltzmann constant
- \( (\rho c)_p \) heat capacities of nanofluid
- \( (\rho c)_p \) effective heat capacity of the nanoparticles
- \( \beta \) Deborah number
- \( \gamma \) chemical reaction parameter

**Subscripts**

- \( \infty \) infinity
- \( w \) sheet surface
REFERENCES


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Fig. 2: Velocity and temperature profile for various values of $A_1$.

Fig. 3: Velocity and temperature profile for various values of $A_2$. 
Fig. 4: Velocity and temperature profile for various values of $\beta$.

Fig. 5: Velocity and temperature profile for various values of $\lambda$. 
Fig. 6: Velocity and temperature profile for various values of $M$.

Fig. 7: Temperature profile for various values of $\theta_w$ when $Pr = 6.2$. 
Fig. 8: Temperature profile for various values of $\theta_w$ when $Pr = 10$.

Fig. 9: Temperature profile for various values of $Nr$. 
Fig. 10: Nanoparticle concentration profile for various values of $\gamma$.

Fig. 11: Temperature profile for various values of $Le$. 

$A_1 = 0.5, A_2 = -1, M = 0.3, \beta = 0.2, \lambda = 0.5, Nb = Nt = 0.1, A' = B' = 0.5, Nr = 0.5, \theta_{w} = 1.2, Le = 10, Pr = 6.2.$

$\gamma = -0.2, -0.1, 0, 0.1, 0.2$

$A_1 = 0.5, A_2 = -1, M = 0.3, \beta = 0.2, \lambda = 0.5, Nb = Nt = 0.1, \gamma = 0.1, A' = B' = 0.5, Nr = 0.5, \theta_{w} = 1.2, Pr = 6.2.$

$Le = 5, 10, 15, 20$
Fig. 12: Nanoparticle concentration profile for various values of $Le$.

Fig. 13: Temperature and Nanoparticle concentration profile for various values of $Nb$. 
Fig. 14: Temperature profile for various values of Nt.

Fig. 15: Nanoparticle concentration profile for various values of Nt.
Fig. 16: Temperature profile for various values of $Pr$.

Table 1. Comparison table for $-\theta'(0)$ (viscous case) with $\beta = \lambda = A_1 = A_2 = \nu_r = \gamma = 0, Nb = Nt = 10^{-6}$.

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Table 2: Values of Skin friction coefficient and Nusselt number for different values of the parameters when $Pr = 6.2, \theta_w = 1.2, \nu_r = 0.5$.

<table>
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<tr>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$M$</th>
<th>$-\sqrt{Re}C_x$</th>
<th>$\frac{Nu_x}{\sqrt{Re}}$</th>
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Table 3: Values of Nusselt and Sherwood number for different values of the parameters when $A_1 = 0.5, A_2 = -1, \beta = 0.2, \lambda = 0.5, M = 0.3.$

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<th>Nb</th>
<th>Nt</th>
<th>$\gamma$</th>
<th>Le</th>
<th>Pr</th>
<th>$\frac{-Sh_x}{\sqrt{Re_x}}$</th>
<th>$\frac{-Nu_x}{\sqrt{Re_x}}$</th>
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