ABSTRACT
The Electrical discharge machining is a widely used precision manufacturing process. The process involves a controlled erosion of electrically conductive materials by initiation of rapid and repetitive spark discharges between electrode tool and work piece, separated by a small gap of about 0.01 to 0.05mm known as spark gap. In the present work, copper and brass are used as tool materials and mild steel EN8 is used as work piece material. The process parameters selected are discharge current (Ip), pulse on time (Ton), pulse off time (Toff) over the response of metal removal rate (MRR). A full factorial design of experiments is used to find the influence of process parameters on metal removal rate (MRR). The experiments are repeated using a copper and brass electrode and the main and interaction effects are plotted. From the experiments it was found that interaction of discharge current and pulse on time, Discharge current are the most influencing factors using brass as electrode. If copper is used as electrode then interaction of discharge current and pulse on time, Pulse off time are the most influencing factors on MRR.

INTRODUCTION
The Electrical Discharge Machine (EDM) process involves a controlled erosion of electrically conductive materials by initiation of rapid and repetitive spark discharge between electrode tool and work piece, separated by a small gap of about 0.01 to 0.05mm known as spark gap[1,7]. This is either flooded or immersed under dielectric fluid. The controlled pulsing of direct current produces the spark discharge between the work piece and tool. Each spark produces enough heat to melt and vaporize a tiny volume of the work piece material leaving a small crater on its surface. The energy contained in each spark is discrete and it can be controlled so that material removal rate, surface finish and tolerance can be predicted[2,5].

EDM has the ability to machine complex shapes in very hard metals. The most common use of EDM is in machining dies, tools and moulds made of hardened steel, tungsten carbide, high-speed steel and other work piece materials that are difficult to machine by “traditional” methods. Because of technical advances in electrode wear, accuracies and speed, EDM has replaced many of the traditional processes [3,4]. Another factor contributing to the growing use of EDM is the expansion of the work envelope, particularly when it comes to heights and tapers.

METHODOLOGY
A Mathematical model is developed to optimize the input parameters for each tool used for machining by Design of Experiments. In the design of experiments, numbers of trails to be conducted are determined by factorial method and design matrix is constructed for both copper and brass tool used. The experiments are carried out as per the design matrix. After getting the design matrix, regression coefficients are calculated. Adequacy of model is tested by fisher test at 5% significance level. Student’s t-test is done for each regression coefficient to check the significance. The final mathematical model is formed after removing non-significant coefficients. Finally Analysis of Variance (ANOVA) is done to find out the percentage contribution of each factor to the metal removal rate[6,8].
Framing a Mathematical Model

The EDM process variables (factors) are identified to develop the mathematical model to predict the MRR. These include pulse on time (\(T_{on}\)), pulse off time (\(T_{off}\)) and pulse current (\(Ip\)). The first order model is assumed with two and three four interactions which can be expressed as

\[
Y=b_0+b_1X_1+b_2X_2+b_3X_3+b_4X_1X_2+b_5X_1X_3+b_6X_2X_3+b_7X_1X_2X_3+b_8X_1X_3+b_9X_2X_3+b_{10}X_1X_2X_3
\]  

(1)

Where \(Y\) represents MRR, \(X_1, X_2, X_3\) represents the coded values of \(T_{on}, T_{off}\) and \(Ip\) respectively; \(b_0, b_1, b_2 \ldots b_{10}\) are regression coefficients of the polynomials to be determined.

Design of Experiments

A two level full factorial design of experiments is adopted for calculating the main and the interaction effects of the three factors at two levels (\(2^3 = 8\) experiments are conducted to fit an equation. The experiments are done for both the tools and the equation is written. After the experimentation, the MRR values are calculated.

**Figure 1:** Samples before and after machining with both the tools

Decoding of Coded Linear Equation

Decoding of linear equation (1) is done by substituting \((a_1-\text{Avg}_1)/\text{VI}_1, (a_2-\text{Avg}_2)/\text{VI}_2\) and \((a_3-\text{Avg}_3)/\text{VI}_3\) in place of \(X_1, X_2, X_3\).

Where \(a_1, a_2\) and \(a_3\) are natural values of factors
\(\text{Avg}_1, \text{Avg}_2\) and \(\text{Avg}_3\) are the average values of the factors
\(\text{VI}_1, \text{VI}_2\) and \(\text{VI}_3\) are the variation intervals.

\[
\text{AVG} = \frac{X_{\text{max}} + X_{\text{min}}}{2}
\]

\[
\text{VI} = \frac{X_{\text{max}} - X_{\text{min}}}{2}
\]

Development of the Model

Design matrix for a given 2-level and 3-factor is generated and the regression coefficients are calculated. Here the number of replications for the response is \(y_1\) and \(y_2\) and average of these is ‘\(y\)’.

Regression coefficients \(b_0, b_1, b_2, b_{12}, b_{23}\) etc are calculated by using the following formula

\[
b_j = \frac{\sum_{i=1}^{N} X_{ij}Y_i}{N}
\]

Where \(N\) is the number of trials

Then Fisher’s test for the adequacy of the model is done

Variance of Reproducibility \((S_y^2) = 2\Sigma (\Delta y)^2/N\)

Where \(\Delta y = (y_i-y)\)

Variance of adequacy \((S_{ad}^2) = 2\Sigma (y_i-y_p)^2/\text{DOF}\)

Where \(y_p\) = predicted response

\(y_p = b_0 x_0 + b_1 x_1 + b_2 x_2 + \ldots\)

Where \(\text{DOF} = \text{degree of freedom} = [N-(k+1)],\)

\(N=\text{number of trails}, k=\text{number of factors}.

F_{\text{Model}} = (S_{ad}^2)/(S_y^2)

For given values of \(f_1\) and \(f_2\), F-table value is found from Fisher table.

Here \(f_1 = N-(k+1)\) and \(f_2 = N\)

If \(F_{\text{Model}} \leq F\)-table, model is adequate in linear form
RESULTS AND DISCUSSION
The results obtained from the experiments as per the design matrix is given below.

<table>
<thead>
<tr>
<th>S. No</th>
<th>Design Matrix</th>
<th>Resultant MRR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X1</td>
<td>X2</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>4</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>7</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>8</td>
<td>+1</td>
<td>+1</td>
</tr>
</tbody>
</table>

CALCULATION OF REGRESSION COEFFICIENTS
The linear equation is formed for both the tools used and regression coefficients are calculated for both the equations. The calculated regression coefficients are shown in the below table.

<table>
<thead>
<tr>
<th>Regression Coefficients</th>
<th>Copper</th>
<th>Brass</th>
</tr>
</thead>
<tbody>
<tr>
<td>b0</td>
<td>24.184</td>
<td>8.828</td>
</tr>
<tr>
<td>b1</td>
<td>1.839</td>
<td>1.947</td>
</tr>
<tr>
<td>b2</td>
<td>-0.821</td>
<td>-0.487</td>
</tr>
<tr>
<td>b3</td>
<td>9.197</td>
<td>-0.322</td>
</tr>
<tr>
<td>b12</td>
<td>1.077</td>
<td>2.921</td>
</tr>
<tr>
<td>b23</td>
<td>12.691</td>
<td>-0.371</td>
</tr>
<tr>
<td>b31</td>
<td>1.81</td>
<td>1.154</td>
</tr>
<tr>
<td>b123</td>
<td>1.524</td>
<td>1.307</td>
</tr>
</tbody>
</table>

Checking the adequacy for copper tool

\[ S_{y}^{2} = 0.013335 \]
\[ S_{ad}^{2} = 0.00 \]
\[ F_{-model} = \frac{S_{ad}^{2}}{S_{y}^{2}} = 0 \]
\[ F_{-table} = 3.8 \]
Since \( F_{-model} \leq F_{-table} \),
Therefore model is adequate in linear form.
The final equation of mathematical model in linear form is
\[ Y_{COPPER} = 24.184 + 1.839X_{1} - 0.821X_{2} + 9.197X_{3} + 12.691X_{1}X_{2} + 1.077X_{1}X_{3} + 1.811X_{2}X_{3} + 1.524X_{1}X_{2}X_{3} \]

Checking the adequacy for Brass tool

\[ S_{y}^{2} = 0.01975 \]
\[ S_{ad}^{2} = 0.00 \]
\[ F_{-model} = \frac{S_{ad}^{2}}{S_{y}^{2}} = 0 \]
\[ F_{-table} = 3.8 \]
Since $F_{model} \leq F_{table}$,
Therefore model is adequate in linear form.
The final equation of mathematical model in linear form is

$$Y_{BRASS} = 8.828 + 1.947X_1 - 0.487X_2 - 0.322X_3 + 2.921X_1X_2 - 0.371X_1X_3 + 1.154X_2X_3 + 1.307X_1X_2X_3$$

**ANALYSIS OF VARIANCE (ANOVA)**
Analysis of variance is done to find out the percentage contribution of each factor and relative significance of each factor. The ANOVA table for the model when copper and brass is used as tool material is shown below.

<table>
<thead>
<tr>
<th>Factors</th>
<th>% Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Copper</td>
</tr>
<tr>
<td>$X_1$</td>
<td>1.316</td>
</tr>
<tr>
<td>$X_2$</td>
<td>0.262</td>
</tr>
<tr>
<td>$X_3$</td>
<td>32.928</td>
</tr>
<tr>
<td>$X_{12}$</td>
<td>62.698</td>
</tr>
<tr>
<td>$X_{23}$</td>
<td>0.452</td>
</tr>
<tr>
<td>$X_{31}$</td>
<td>1.277</td>
</tr>
<tr>
<td>$X_{123}$</td>
<td>0.904</td>
</tr>
</tbody>
</table>

**Figure 2:** Effect of pulse current on MRR when copper is used as electrode

**Figure 3:** Effect of pulse off time on MRR when brass is used as electrode

**CONCLUSIONS**
From the experimental results it was found that interaction of pulse current and pulse on time are the most influencing factors on MRR using copper as electrode material. If brass is used as electrode material then the
MRR is decreased with the increase of pulse on time and pulse off time. These are the most influencing factors on MRR in addition to current.

REFERENCES